## The Moon-moon Challenge

Can you get a satellite to orbit the Earth's moon?

The planetarium model will not make this easy, as the interface is designed to help you put moons around planets, not moons around moons!

Neverthertheless, there is nothing stopping you testing this out. The model doesn't care about the interface, the body will behave according to the gravity it sees.

When setting up a moon around a planet, you provide the moon a position with respect to Lunar Distances (LD), and a relative velocity in terms of the circular orbit velocity.

Fig 1 - interface for setting position of moons and relative velocity.

## Position (LD)

## Lunar Distance

## Orbit Velocity (relative*)

* Relative to circular orbit speed

| -1.063 | $\hat{v}$ | 0.096 | $\hat{v}$ | 0 |
| :--- | :--- | :--- | :--- | :--- |$\quad \hat{v}$

1.0673LD, $5.16^{\circ}$ (altitude) $180^{\circ}$ (azimuth)

$949.95 \mathrm{~ms}^{-1} 0^{\circ}$ (altitude) $90^{\circ}$ (azimuth)

The circular orbit velocity is defined by the formula:

$$
V_{c}=\sqrt{\frac{G M_{e}}{d}}
$$

G - universal gravity constant $6.6743 \times 10^{-11} \mathrm{~m}^{3} \mathrm{~kg}^{-1} \mathrm{~s}^{-2}$
$\mathrm{M}_{\mathrm{e}}$ - Mass of planet (kgs)
d - distance from centre of planet to center of satellite (m)

Note: The Lunar distance unit is $384,402,000$ metres.

Furthermore, you will need to set the velocity of the satellite to be with respect to the velocity of the moon, but the interface will give you the circular orbit velocity with respect to the Earth. (Which is not helpful!)

The following formula may help. It converts the velocity for you, giving you the relative circular orbit velocity with respect to the Earth you require to give you a circular orbit velocity for the moon:

Fig 2 - Earth on the left, and moon on the right and the satellite to the right of the moon


$$
V R_{s}=V R_{m} \sqrt{\frac{d_{2}}{d_{1}}} \pm \sqrt{\frac{M_{m} d_{2}}{M_{e} d_{0}}}
$$

$\mathrm{VR}_{\mathrm{s}}$ - Relative velocity of the satellite (with respect to circular orbit velocity about Earth)
$\mathrm{VR}_{\mathrm{m}}$ - Relative velocity of the moon (with respect to circular orbit velocity about Earth)

But as we need to work in three dimensions, take note of which direction you are applying this to. For a circular orbit to work, the velocity needs to be perpendicular to the position!

The simplest setup is to line up the Earth, moon and satellite in one line, like in Fig 2.
In this set up $\mathrm{d}_{1}+\mathrm{d}_{0}=\mathrm{d}_{2}$
This way we don't need to use any trigomometry or pythagorus, and we can just use the numbers from the above formula. You can find the Earth and Moon masses from the current set up.

Try a certain satellite distance from the moon in Lunar distances and calculate the relative circular velocity value you will need that should give it a circular orbit about the Moon.

But which box do you enter this in? The $\mathrm{X}, \mathrm{Y}$ or Z direction? Or all?

Check how the velocity has been set up for the moon, which direction is the moon's initial velocity been set up as?

Use that same direction for the satellite's initial velocity.

## What about the position of the satellite?

If you look at how the position of the moon has been set up initially, it is not perfectly in the $\mathrm{X}-\mathrm{Z}$ orbital plane, in fact it is out by $5.16^{\circ}$

So to have the satellite line up, you will need to do the same. You have already determined the distance you want it as $\mathrm{d}_{2}$ (fig 2), but you will need to define that position now as a vector with the same direction as the position of the moon.

Set the X position as the cosine of $5.16^{\circ}$ and Z position as the sine of $5.16^{\circ}$.

Or, cheat by moving the moon into the orbital plane? (it might make things easier!)

Make sure you apply your changes.

Before you start the simulator, go to the information panel for the Earth, select the satellite and bring up the information for the satellite. Turn on the orbit trace and check that the orbit is close to circular. If you have used the formula correctly, the answer might be disappointing - the orbit is not quite circular, and that is because the formula is not quite right, but it is quite close. The formula makes an assumption about the Earth - Moon orbit which is a bit wrong. Can you guess what this might be?

Run the simulation - what happens?

Try different distances, what is the most number of orbits of the moon you can get your satellite to do?

Think outside the square, what ways can you make our satellite orbit more stable? (Use your Omnipotent powers if you have to!)

